XXVII

FATIGUE CRITERIA

Two fatigue criteria are formulated here in order that both relatively low-strength ductile materials and high-strength, more brittle materials may be used in one design. The intention is to use high-strength steels as liner materials and lower strength ductile steels for the outer cylinders in order to prevent catastrophic brittle failure.

Fatigue Criterion for Ductile Outer Cylinders

From both torsion and triaxial fatigue tests on low-strength steels (120 to 150 ksi ultimate strength) conducted by Morrison, Crossland, and Parry[35] it is concluded that a shear criterion applies. Therefore, a shear theory of failure is assumed for outer rings made of ductile steel.

To formulate a fatigue relation, the semirange in shear stress and the mean shear stress are needed. These stresses are defined as

$$S_r = \frac{S_{max} - S_{min}}{2}$$

$$S_m = \frac{S_{max} + S_{min}}{2} \qquad (6a, b)$$

respectively.

A linear fatigue relation in terms of shear stresses is assumed. This relation is

$$\frac{S_r}{S_e} + \frac{S_m}{S_u} = 1, \text{ for } S_m \geqq 0 \quad ,$$

where S_e is the endurance limit in shear and S_u is the ultimate shear stress. For $S_u = 1/2\ \sigma_u$, where σ_u is the ultimate tensile stress, this relation can be rewritten as:

$$\frac{S_r}{S_e} + \frac{2S_m}{\sigma_u} = 1, \ S_m \geqq 0 \qquad (7)$$

The stresses S_r and S_m given by Equations (6a, b) can be calculated from elasticity solutions. In order to employ the fatigue relation (7) for general use, it is assumed that S_e can be related to S_u. This is a valid assumption as shown by Morrison, et al[35]. Referring to Reference (35), the ratio S_e/S_u can be established. Table XLI lists some fatigue data and results of calculation of S_e from Equation (7).

From Table XLI is is evident that fluid pressure contacting the material surface has a detrimental effect on fatigue strength; the endurance limit S_e for unprotected triaxial fatigue specimens is lower than that for torsional specimens. However, protection of the bore of triaxial specimens increases S_e under triaxial fatigue to a value equal that for torsional fatigue. Since in the high-pressure containers, outer cylinders are subject to interface contact pressures and not to fluid pressures, it is assumed that the data for a protected bore in Table XLI are applicable in the present analysis. Therefore, the following relation between S_e and σ_u is assumed:

$$S_e = \frac{1}{3}\sigma_u \tag{8}$$

Substitution of Relation (8) into (7) gives

$$3S_r + 2S_m = \sigma, \text{ where } \sigma \leqq \sigma_u \tag{9}$$

Equation (9) now has a factor of safety, σ_u/σ, and can be expected to predict lifetimes of 10^6 cycles and greater for ductile steels based upon the linear fatigue relation and available fatigue data. (Of course, stress concentration factors due to geometrical discontinuities or material flaws would reduce the expected lifetime.)

TABLE XLI. TORSIONAL AND TRIAXIAL FATIGUE DATA ON VIBRAC STEEL[(a)]

Test	Stresses, psi				
	σ_u	S_r	S_m	S_e	S_e/σ_u
Torsion	126,000	43,700	0	43,700	0.347
	149,000	52,900	0	52,900	0.354
Triaxial (unprotected bore)	126,000	20,900	20,900	31,300[(c)]	0.248
	149,000	26,300	26,300	40,600	0.273
Triaxial[(b)] (protected bore)	126,000	26,500	26,500	45,900	0.363

(a) From Reference (35). Composition of this steel in weight percent is 0.29 to 0.3 C, 0.14 to 0.17 Si, 0.64 to 0.69 Mn, 0.015 S, 0.013 P, 2.53 to 2.58 Ni, 0.57 to 0.60 Cr, 0.57 to 0.60 Mo.

(b) The bore of the cylindrical specimens was protected with a neoprene covering.

(c) S_e for the triaxial tests is calculated from Equation (7).

Fatigue Criterion for High-Strength Liner

Triaxial fatigue data on high-strength steels ($\sigma_u \geqq 250$ ksi) are not available. Fatigue data in general are very limited. Therefore, a fatigue criterion for high-strength steels under triaxial fatigue cannot be as well established as it was for the lower strength steels. The high-strength steels are expected to fail in a brittle manner. Accordingly, a maximum tensile stress criterion of fatigue failure is postulated.

Because fatigue data are limited while tensile data are available the tensile stresses $(\sigma)_r$ and $(\sigma)_m$ are related to the ultimate tensile strength by introduction of two parameters α_r and α_m. These are defined as follows: